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Indentation creep measurements using a Rockwell hardness tester

V. R. HOWES
Department of Applied Physics, School of Physics, University of New South Wales, Kensington, 2033, Australia

A Rockwell superficial hardness tester has been used to continuously monitor hardness under load with time. An analysis of the Rockwell test, that showed the relationship between the measured parameters and the on-load hardness and elastic recovery has been quantitatively verified.

Low homologous temperature, high stress, indentation creep in the macrohardness range has been measured for aluminium, copper and mild steel, but was not detectable for brass and silver steel. The stress relaxation was no more than 10% and appeared to follow an exponential decay to an equilibrium value. The difference between normal uniform tensile creep and indentation creep, and explanations for the latter, have been discussed, and the measured relaxation time has been related to a ratio of elastic and creep indentation material constants.

Bearing in mind the world-wide availability of Rockwell hardness testers, it is suggested that this extension to creep testing, and possibly fatigue testing, may be a useful development for industrial non-destructive quality control and specification checking of materials and components.

1. Introduction

The Rockwell hardness test is widely used in industry for quality control and specification checking. It is instrumentally simple, direct reading and fast. The test is described by Lysaght [1] and is standardized by the ASTM [2]. The instrument indicates the indenter penetration and registers an arbitrary hardness number that decreases as the indentation depth increases. The technical advantages of the technique are that the deformation is measured directly as a penetration depth and optical measurement of indent size is unnecessary, the penetration is measured at nearly zero load so that the instrument compression error is negligible, and the application and retention of a minor load holds the indenter in position and reduces the secondary effect of the state of the specimen surface. It is little used in materials research studies because the Rockwell hardness number is of remote physical significance and the measurement incorporates the recovery that may occur upon load removal, so that hardness, as defined by the ratio of indentation load to the deformation area under load, is not precisely measured. For research, the Brinell or Vickers type of indentation hardness test is usually preferred, however, during a study of indentation creep using a method of continuously recording the indent contact area by electrical resistance measurement [3], it was realized that a similar continuous recording was potentially possible with a Rockwell hardness tester. In a subsequent consideration of the operation of the Rockwell test it became apparent that the test could be used to measure not only the hardness of a material, but also its elastic and creep properties.

Indentation creep is a form of high-stress creep testing in which indentation deformation is measured as a function of dwell time of a loaded indenter. The phenomenon has been reviewed by Walker [4] and an analysis of microhardness indentation creep in terms of standard creep mechanisms has been given by Atkins, Silverio and Tabor [5]. The test is currently being used to study high and low temperature creep of ultra high strength materials [6]. For metals at low homologous temperatures Walker [4] comments that results of indentation creep testing have been varied and controversial. The main difficulty in making creep measurements with the usual Brinell or Vickers type of hardness tester is that the deformation is measured after load removal. This means that an extensive series of indentations has to be done for each creep test, and small hardness variations due to creep may be submerged in the not insignificant statistical variations of hardness measurements over a test specimen. Although the Rockwell hardness number also relates to the unloaded indentation, the Rockwell test includes information on the indenter penetration under load. Thus creep can be continuously followed for any one indentation.

2. Analysis of the Rockwell test

The indenter is first applied to the sample with a small fixed pre-load (L_P), the consequent penetration (Z_P) is

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not measured. The main load (L_M), selected from a given range, is then applied. At this point the instrument registers a number of divisions (N_M) related to the indenter penetration (Z_M) occurring as the load increases from L_P to L_T which is equal to $L_P + L_M$ (e.g. for a simple dial gauge instrument, the needle moves around N_M divisions), but this plays no part in the Rockwell hardness measurement. The major load is then removed and the instrument registers another number of divisions in the opposite direction (N_R), related to the recovery (R_M) of the sample material as the load is reduced by L_M from L_T to L_P (for a simple dial gauge instrument, the needle moves back N_R divisions). The resultant displacement, given by $(N_M - N_R)$, is what is used to derive the Rockwell hardness number (for a simple dial gauge instrument, the scale is arranged and set so that the needle indicates this number directly). It can be seen that for the indentation

$$Z_M = C_S N_M - K_M \quad (1)$$

where C_S is the sensitivity constant of the instrument in micrometres per scale division, and K_M the elastic compression of the instrument under load L_M (the pressure shaft and the indenter.holder of the Rockwell tester are compressed and so shortened). For the recovery in the opposite direction

$$R_M = C_S N_R - K_M \quad (2)$$

It may be noted that

$$Z_M - R_M = C_S(N_M - N_R) \quad (3)$$

which represents the final penetration of the indenter after the major load has been removed, and this is independent of the instrument compliance under load (K_M has been eliminated).

Standard Rockwell hardness testers have a pre-load of 10 kg and a major load range of 60 to 150 kg, and each division or point of hardness is the equivalent of 2 μm in indentation ($C_S = 2$ μm div^{-1}). There is also an ASTM standardized Rockwell superficial hardness test [2] in which the pre-load is 3 kg and the major load range is 15 to 45 kg. With this tester the indents are smaller and the sensitivity has been increased so that each division or point of hardness corresponds to a difference in depth of penetration of 1 μm ($C_S =$ 1 μm div^{-1}). In the Rockwell test the indenter is either a spherical steel ball or a 120° diamond cone with a rounded point, but the instrument can just as well be fitted with any indenter if required. By registering the deformation after the main load has been removed and recovery has occurred, the Rockwell number is related to what has been called elsewhere the surface resistance to damage [7], i.e. the indentation load divided by the permanent area of deformation remaining after load removal. For most metals the difference between this and hardness may not be significant, but for high strength metals, ceramics and polymeric materials the difference can be significant and measureable.

The penetration Z_M in Equation 1 is clearly related to the hardness of the indented material. Defining hardness (H) as load (L) divided by the projected area of the indentation under load, which may be taken as $C_I Z_L^2$ where Z_L is the indent depth and C_I a geometrical constant for a cone or pyramid indenter, then

$$Z_M = Z_T - Z_P = (L_T/C_I H_T)^{1/2} - (L_P/C_I H_P)^{1/2} \quad (4)$$

so that from Equations 1 and 4 we have,

$$H_T^{-1/2} = C_S C_I^{1/2} L_T^{-1/2} N_M - K_M C_I^{1/2} L_T^{-1/2} + H_P^{-1/2} L_P^{1/2} L_T^{-1/2} \quad (5)$$

This equation relates hardness to N_M, but it should be noted that K_M is a function of load which may or may not be simply linear. If the approximation is made that $C_S N_M \gg K_M$ in Equation 1, which is likely to be true for other than very hard materials, then the K_M term can be neglected. Assuming H is constant ($H_P = H_T = H$) then

$$H \sim [(L_T^{1/2} - L_P^{1/2})/C_I^{1/2} C_S N_M]^2 \quad (6)$$

so that the hardness can be measured approximately.

For the variation of H_T with time (t) at constant load we have from Equation 5

$$H_{T_t} = 1/(A N_{M_t} + B)^2 \quad (7)$$

where A and B are constants. A is equal to $(C_S C_I^{1/2} L_T^{-1/2})$ and the value is known from the instrument and indenter specifications. B can be found experimentally by measuring the final size of the indent by microscope at the end of the test, so that the final value of hardness (H_{T_f}) can be directly calculated. Then B equals $(H_{T_f}^{-1/2} - A N_{M_f})$. Thus the indentation creep can be accurately followed for one indentation. The experimental errors in measuring the changing H_{T_t} (i.e. the changing N_{M_t}) are much smaller than the statistical errors in measuring the mean hardness obtained from a number of indentations.

The relationship of R_M in Equation 2 to the recovery properties of the indented material is a more difficult problem. It has been observed that where recovery occurs, it is greater in depth than in the surface dimensions of indentations [8]. Stilwell and Tabor [9] showed that upon subsequent reloading at an indentation, the deformation retraces the unloading path to a good approximation. This suggested that the unloading occurs elastically to a first approximation, and could be related to an elastic reloading, which in turn may be related to the loading of a rigid indenter on to a completely elastic material. Sneddon [10] derived the mean contact pressure (P_E) for the contact of a rigid cone with an elastic half-space (Young's modulus E and Poisson's ratio ν) as

$$P_E = E \cot\theta/2(1 - \nu^2) = 0.5\varepsilon\cot\theta \quad (8)$$

where θ is the indenter semi-angle and ε is $E/(1 - \nu^2)$. Lawn and Howes [11] have shown that, allowing for residual forces and using reloading mechanics for the essentially reversible unloading half-cycle, to a first approximation

$$(Z_R/Z)^2 = 1 - 2\tan\theta(H/\varepsilon) \quad (9)$$

where Z_R is the residual depth ($Z_R = Z - R$). Thus we may write

$$R_M = R_T - R_P$$
$$= Z_T\left[1 - \left(1 - 2\frac{H_T}{\varepsilon}\tan\theta\right)^{1/2}\right] - R_P \quad (10)$$

From Equation 2 for R_M and substituting for Z_T as in Equation 4, we have from Equation 10

$$\left[1 - \left(1 - 2\frac{H_T}{\varepsilon}\tan\theta\right)^{1/2}\right]H_T^{-1/2} =$$
$$C_S C_I^{1/2} L_T^{-1/2} N_R - K_M C_I^{1/2} L_T^{-1/2} + R_P C_I^{1/2} L_T^{-1/2} \quad (11)$$

For materials where the elastic modulus is much greater than the hardness so that the second term in the square root is less than one, this may be expanded to give the approximation

$$(H_T^{1/2}/\varepsilon)\tan\theta \sim C_S C_I^{1/2} L_T^{-1/2} N_R - K_M C_I^{1/2} L_T^{-1/2} + R_P C_I^{1/2} L_T^{-1/2} \quad (12)$$

Equation 11 or 12 is the equivalent of Equation 5, in this case relating the material properties ratio $(H^{1/2}/\varepsilon)$ to N_R. Thus information on the elastic properties of indented materials can be obtained by measuring N_R, however, since the value of $(H_T^{1/2}/\varepsilon)$ is often very small, the K_M term cannot be taken as negligible here. In fact for plastic materials where recovery is very small, it is the R_M term in Equation 2 that may be neglected to give the approximation

$$K_M \sim C_S N_R \quad (13)$$

where N_R is not measurably different for different materials, the material recovery being masked by the instrument compressibility.

3. Experimental verification of the analysis

A Rockwell superficial hardness tester was used with loads of 15, 30 and 45 kg automatically selected. The specification of C_S was 1 μm per division and the values of N_M and N_R for indentations made were read directly from the needle movements on the dial indicator. The specimens, tested at room temperature, were parallel sided, mechanically polished, unmounted polycrystalline samples of aluminium (A), copper (C), brass (B), mild steel (M), and silver steel (S). The measured hardnesses and the elastic properties obtained from data tables are given in Table I. Measuring the size of the indents by microscope after indentation enabled the hardness to be obtained independent from the readings of the Rockwell tester. In order to approach the on-load hardness as closely as possible, a standard Vickers 136° diamond pyramid indenter was used, for which recovery at the diagonal corners is minimal.

Tests were made at the three loads available and Fig. 1 shows the results. N_M is clearly inversely related to H_v. N_R values appear to increase with $H_v^{1/2}/\varepsilon$ (from Equation 12), but the influence of load L_T is much stronger. Of course the compression K_M also increases significantly with load.

TABLE I Hardness and elastic properties of materials used

Material	Symbol	H_T (Vickers) ($kg\,mm^{-2}$)	E ($kg\,mm^{-2}$) $\times 10^3$	ν	H_T/ε $\times 10^{-3}$
Aluminium	A	31	7.15	0.34	3.8
Copper	C	93	12.5	0.34	6.6
Brass	B	115	11	0.345	9.2
Mild steel	M	205	21	0.3	8.9
Silver steel	S	770	21	0.3	33.5

E is the Young's elastic modulus, ν Poisson's ratio and $\varepsilon = E/(1 - \nu^2)$

To allow a more quantitative verification of the analysis, the unknown K_M term can be avoided by referring to Equation 3. Thus from Equations 3, 4 and 10 we have

$$(N_M - N_R) = C_S^{-1} C_I^{-1/2} H_T^{-1/2} \times \left(1 - 2\frac{H_T}{\varepsilon}\tan\theta\right)^{1/2} L_T^{1/2} - C_S^{-1}(Z_P - R_P) \quad (14)$$

For any one material the last term is a constant as L_T varies, so that the relationship between $(N_M - N_R)$ and $L_T^{1/2}$ should be linear, as shown in Fig. 2 for all the materials tested (note that for $(N_M - N_R)$ equal to zero, all plots approach $L_T^{1/2}$ approximately equal to $L_P^{1/2}$ as would be expected). The slopes of these straight lines (S) multiplied by $C_S C_I^{1/2}$ (where C_S is taken as 0.001 mm div^{-1} and C_I is $7^2/2$ for a Vickers pyramid indenter) should be equal to $H_T^{-1/2}$ $[1 - 2(H_T/\varepsilon) \times \tan\theta]^{1/2}$. By calculating the values of the latter term from the data in Table I and taking $\tan\theta$ equal to 7/2

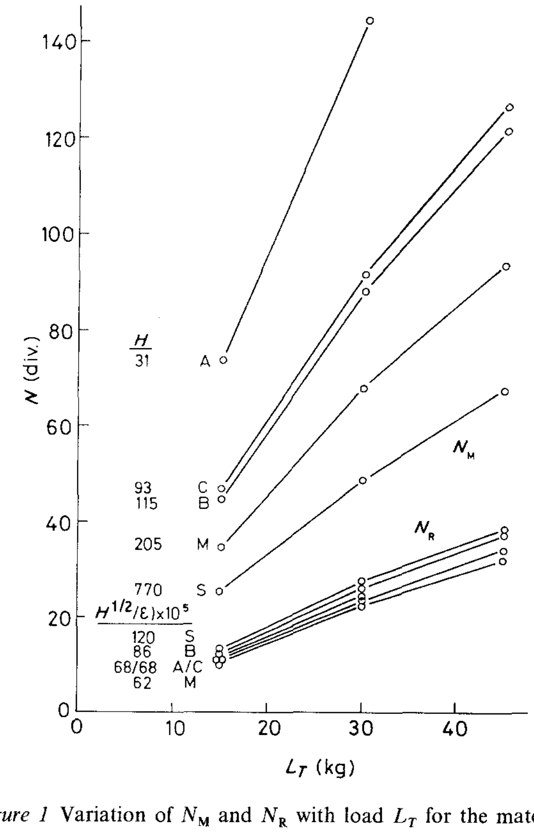

Figure 1 Variation of N_M and N_R with load L_T for the materials tested (see Table I). For the N_M plots values of H in kg mm^{-2} are given, and for the N_R plots values of $H^{1/2}/\varepsilon$ in kg$^{-1/2}$ mm are given.

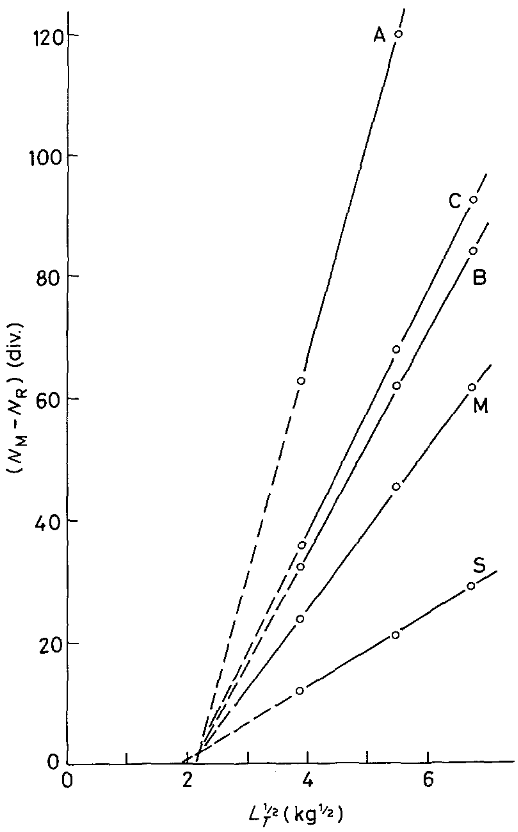

Figure 2 Plots of $(N_M - N_R)$ against $L_T^{1/2}$ showing a linear relationship for the materials tested.

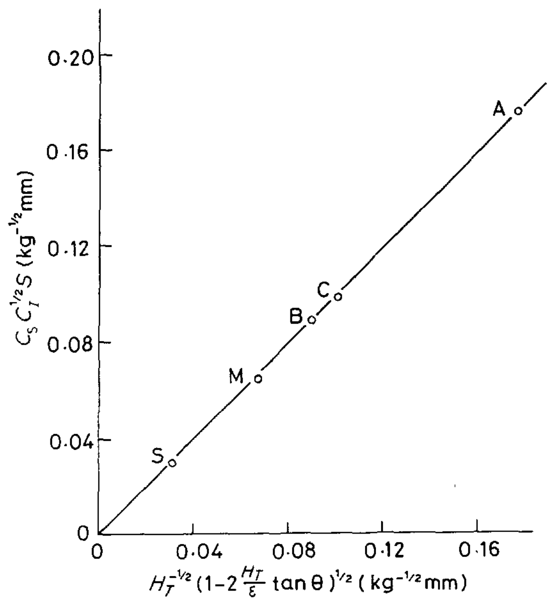

Figure 3 Graph showing the equivalence between $C_S C_T^{1/2} S$ (where S values are the slopes from Fig. 2) and $H_T^{-1/2}[1 - (2H_T/\varepsilon)\tan\theta]^{1/2}$.

for a Vickers indenter, this equality is confirmed in Fig. 3.

It may be noted that an estimate can be made of the compression of the instrument (K_M) from Equation 11 if it is assumed that R_P is small enough to be neglected. Assuming K_M equals $C_C L_M$ where C_C is taken as a compressibility parameter, calculations gave C_C varying from 0.8 to 1.1 $\mu m\,kg^{-1}$ for the Rockwell superficial hardness tester used. The value decreased as the load increased – probably due to an initial settling-in effect.

4. Indentation creep results

The variation of hardness with time was studied at room temperature for the same five materials as used above. Tests were carried out with loads of 15 and 45 kg (except for aluminium which was too soft for indentation at 45 kg so 30 kg was used). To minimize the complexity of indentation deformation, a Rockwell 120° diamond cone indenter was used (so that the deformation was at least radially symmetric). Equation 7 was used to obtain H_{T_t}.

Fig. 4 shows the variation of hardness with time for aluminium (A), at loads of 15 and 30 kg, the error bars indicate the experimental error for the calculated H_{T_t} values. It is seen that H_{T_t} decreased to an apparently equilibrium value where the experimental error was bigger than any measurable further decrease, and the variation was nearly the same for tests at the two different loads. Results for copper (C), brass (B) and mild steel (M) at loads of 15 and 45 kg are shown in Fig. 5. Copper and mild steel show a similar type of H_{T_t} decrease as found for aluminium, but for brass no change in H_{T_t} was observed. Tests on silver steel (S), with a much higher hardness, also showed no measureable change in H_{T_t}. Table II summarizes the changes found in these experiments. It should be noted that whereas the measured hardness changes with time are much greater than the experimental errors in measuring H_{T_t} in any one test, the differences

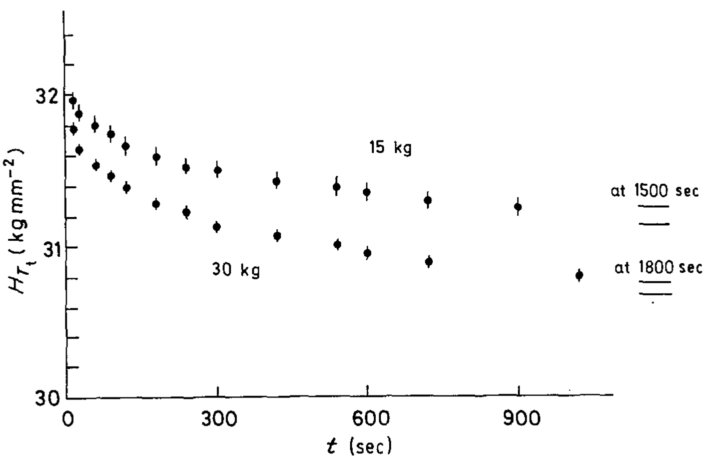

Figure 4 Variation of hardness with time for aluminium, measured with a 120° diamond cone indenter at loads of 15 and 30 kg.

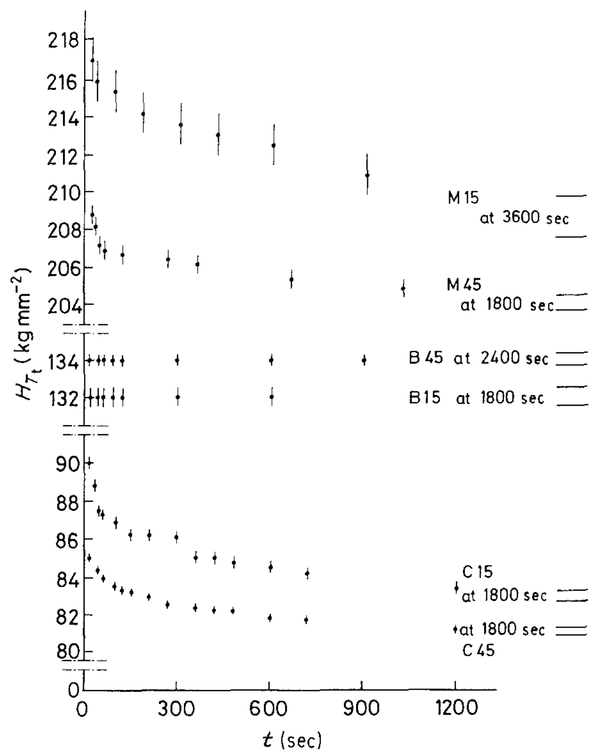

Figure 5 Variation of hardness with time for copper (C), brass (B) and mild steel (M), measured with a 120° diamond cone indenter at loads of 15 and 45 kg.

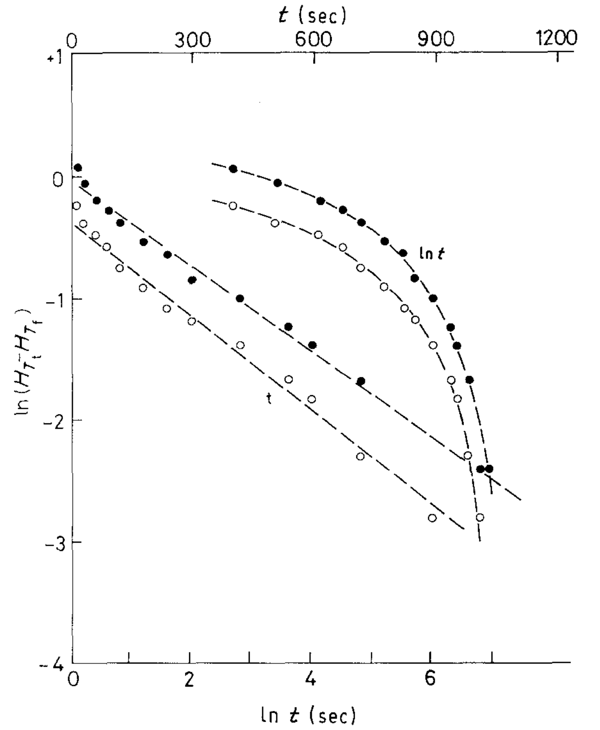

Figure 6 Plots of ln $(H_{T_t} - H_{T_f})$ against time (t) and against ln t for aluminium at indentation loads of 15 (○) and 30 (●) kg.

in hardness for tests at different loads are within the usual statistical error involved in measuring a mean hardness by making several indentation tests.

The measured decreases are all less than 10% and since the mean stress or pressure on the material at the indentation decreases as the hardness decreases, it seems likely that an equilibrium value of hardness will be asymptotically approached, representing a threshold stress below which the mechanisms of low temperature indentation creep cease to operate. From the results, the final measured value of H_{T_t} (H_{T_f}) may be taken as an experimental approximation to this equilibrium hardness. In order to find out whether the observed decreases were associated with an inverse power relationship or an exponential decay it was decided to plot $\ln(H_{T_t} - H_{T_f})$ against ln t and against t. Fig. 6 shows the results for aluminium. It appears that the observed decrease after a brief initial transient, is an exponential decay rather than an inverse power effect. This was also found to be so for the hardness decreases measured for copper and mild steel (see Fig. 7). The slopes of these exponential plots are approximately the same for the tests at the two different loads for each material, and from the slope values a characteristic "relaxation" time (γ) can be calculated. These are given in Table III.

TABLE II Hardness decreases with time obtained for the materials studied

Metal	Homologous temperature	Load (kg)	H_{T_f} (kg mm^{-2})	Maximum decrease in H_T measured (%)
A	0.32	15	31.2	3
		30	30.7	4
C	0.22	15	83	9
		45	81	7
B	0.25	15	132	0
		45	134	0
M	0.17	15	209	4
		45	204	3
S	0.17	15	770	0
		45	910	0

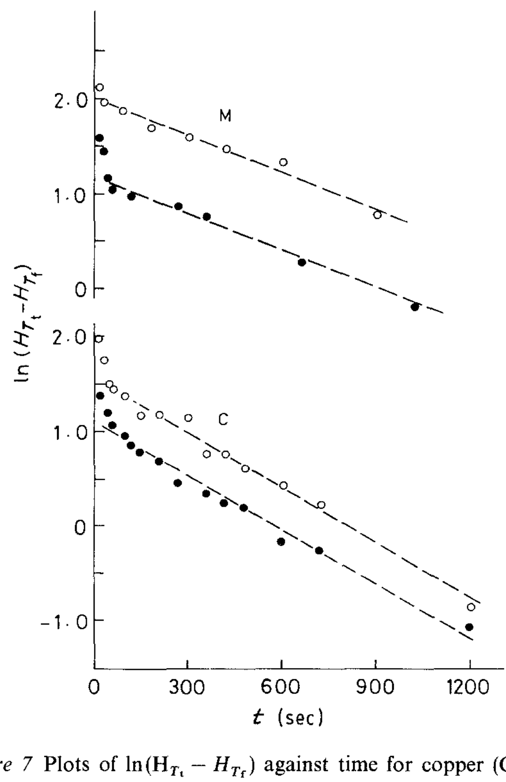

Figure 7 Plots of $\ln(H_{T_t} - H_{T_f})$ against time for copper (C) and mild steel (M) at indentation loads of 15 (○) and 45 (●) kg.

TABLE III Measured relaxation times

Metal	A	C	B	M	S
$\gamma \pm 50$ (sec)	400	550	∞	850	∞

5. Discussion

The main aim of this work has been to establish that the Rockwell hardness tester may be used to simply and accurately study the phenomenon of indentation creep in the macro-indentation range. Where this has been observed, the homologous temperatures have been low ($< 1/3$, see Table II) – well below the temperatures at which normal uniform tensile creep is observed for these materials. The measured indentation hardness decreases ($< 10\%$) at these homologous temperatures are much smaller than the decreases observed by Atkins *et al.* [5] for homologous temperatures above 1/2 (e.g. a decrease from 8.5 to $2.9\,\mathrm{kg\,mm^{-2}}$ for single crystal tin at room temperature). The deformation occurring at an indentation is locally intense and complex, it is not unlikely that the creep or stress-relaxation occurring in this region of intense deformation is significantly different from that in a uniform tensile creep test. Thus the test is relevant to high-stress low-temperature creep problems. The test may be extended to a higher temperature range by the relatively simple addition of a hot stage to the instrument, and it is particularly useful for specifically testing localized areas of components, designed for and subjected to high stresses. Bearing in mind the wide-spread availability of Rockwell hardness testers throughout the world, this extension of the Rockwell instrument to creep testing could be a useful development for industrial non-destructive quality control and specification checking of materials and components.

Previous studies of indentation creep at temperatures above $T_M/2$ (where T_M is the melting point on the absolute scale), have indicated an inverse power law relationship between H and t (see [5]). It was for this reason that $\ln(H_{T_t} - H_{T_f})$ was first plotted against $\ln t$ (Fig. 6). The indication of an exponential decay for the results obtained (Figs 6 and 7) was unexpected and invites some comment. The main problem with the analysis of indentation creep is that although the deformation increases, the overall strain for a cone or pyramid indentation is constant [8], and the mean stress or pressure (L/A) decreases with time! For this reason it is proposed to follow the approach described by Cottrell [12] for stress relaxation at a bolt under constant strain. For indentation the constant strain (I) may be taken as

$$I = I_E + I_P + I_C \quad (15)$$

where I_E is the elastic component of the strain that may in turn be taken as ($C_E P$), where C_E is an elastic proportionality constant characteristic of an indentation in the material and P is the mean indentation pressure or stress (I_E is reversible so that it decreases as P decreases), I_P is the plastic component of the strain (I_P is not reversible so that it is constant as P decreases with time), I_C is the time dependent creep component of the strain. If we suppose that there is a threshold stress (P_R) below which the relaxation mechanism, whatever it might be, does not operate, then I_C will be a function of ($P - P_R$) and time t. Taking the simplest relationship $C_C(P - P_R)t$ as a first approximation, where C_C is a creep constant characteristic of an indentation in the material, then from Equation 15 we have

$$\dot{I} = C_E\dot{P} + O + C_C(P - P_R) = 0 \quad (16a)$$

and solving this differential equation gives

$$\ln(P - P_R) = -(C_C/C_E)t + \ln(P_O - P_R) \quad (16b)$$

or putting H_{T_t} for P and H_{T_f} for P_R:

$$(H_{T_t} - H_{T_f}) = (H_{T_0} - H_{T_f})\exp(-t/\gamma) \quad (16c)$$

which represents an exponential decay with relaxation time γ equal to (C_E/C_C). This derivation involves some considerable simplifications, but nevertheless conforms to the experimental results obtained in this study, and gives a significance to the values of γ in Table III – it is the ratio of indentation elastic and creep material constants (C_E/C_C) which appears to be greater for mild steel than for copper or aluminium, and for brass and silver steel the implication is that C_C approaches zero.

The difference between previous results and the results described here may be due to the former being for predominantly temperature activated creep and the latter being for predominantly stress activated creep. Also, as stated above, the materials tested were all polycrystalline. The grain sizes were less than 50 μm, significantly smaller than the macrohardness indentations made, so the indentations themselves were also polycrystalline. One aspect of an exponential decay is that it is often found for viscoelastic deformation of polymeric materials, and it is tempting to compare the time-dependent untangling of the macromolecules in a polymer with the relaxation untangling of dislocations – by stress induced recovery processes – from the local intensely deformed zone around an indentation in an atomic material. Whatever the recovery processes involved it would seem that they are inhibited more by the microstructures of brass and silver steel than by those of copper and mild steel.

Finally, it is to be noted that the indentation creep experiments described are initial results. Further work with the Rockwell superficial hardness tester with different indenters and an extended range of materials, could be followed by experiments using a standard Rockwell hardness tester involving much greater indentation loads, and, therefore, sizes. The effect of variations in material grain size and other microstructural differences should be investigated to throw more light on the possible mechanism of creep. The metals tested exhibit little elastic recovery after indentation (the H/ε ratios are small – see Table I); it would be interesting to test some materials with high elastic recovery characteristics to see if these could be measured by using the analysis of the Rockwell test involving N_R given above. Also, the Rockwell test is similarly suited to repeated cyclic indentation testing – cyclic loading between the pre-load and the main load avoids the problem of constancy of location under repeated indentation conditions. A program on indentation dynamic fatigue testing has already been started.

6. Conclusions

It has been demonstrated that a Rockwell hardness tester has the potential to measure the scientific indentation hardness (L/A_L) and the elastic recovery properties of materials, and, in particular, can be used to quantitatively study indentation creep.

Indentation creep, in the macrohardness range at low homologous temperatures, has been found for aluminium, copper and mild steel, but was not detected for brass and silver steel.

The observed creep was no more than 10% and appeared to follow an exponential decay of hardness to an equilibrium value. The measured relaxation times were similar for different indentation loads but different for different materials.

The difference between normal uniform tensile creep and indentation creep, and explanations for the latter, have been discussed. The measured relaxation time has been related to a ratio of elastic and creep material constants associated with indentation.

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References

1. V. E. LYSAGHT, in "Indentation Hardness Testing" (Reinhold, New York, 1949).
2. ASTM STP18 (American Society for Testing and Materials, Philadelphia).
3. L. WIECZOREK, V. R. HOWES and H. J. GOLDSMID, *J. Mater. Sci.* **21** (1986) 1423.
4. W. W. WALKER, "The Science of Hardness Testing and Its Research Applications", edited by J. W. Westbrook and H. Conrad (American Society of Metals, Ohio, 1973) p. 258.
5. A. G. ATKINS, A. SILVERIO and D. TABOR, *J. Inst. Met.* **94** (1966) 369.
6. C. A. BROOKS, R. M. HOOPER and J. E. MORGAN, *Eur. Appl. Res. Rep.* **7** (1987) 278.
7. V. R. HOWES and P. WALL, *J. Appl. Polym. Sci.* **33** (1987) 1125.
8. D. TABOR, in "The Hardness of Metals" (Clarendon, Oxford, 1951).
9. N. A. STILWELL and D. TABOR, *Proc. Phys. Soc. London* **78** (1961) 169.
10. I. N. SNEDDON, *Camb. Phil. Soc.* **44** (1948) 492.
11. B. R. LAWN and V. R. HOWES, *J. Mater. Sci.* **16** (1981) 2745.
12. A. H. COTTRELL, in "The Mechanical Properties of Matter" (Wiley, New York, 1964) p. 336.

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